ISSN:0975-9646

Jeyabharthi et al, / (IJCSIT) International Journal of Computer Science and Information Technologies, Vol. 5 (3) , 2014, 4536-4540

Discriminants of Brauer Algebra

Dr.Jeyabharthi,
Associate Professor
Department of Mathematics,
Thiagarajar Engineering College, Madurai.

J.Evangeline Jeba,
Assistant Professor
Department of Mathematics,
Lady Doak College, Madurai.

Abstract: In this work we study how to compute the brauer algebra discriminant and also define a matrix $Z_{m,k}(x)$.

INTRODUCTION:

In the beginning of 20^{th} century invariant theorists began to study the commuting algebras of the tensor powers of defining representations for the classical groups G = Gl(n,C), Sl(n,C), O(n,C), So(n,C) and Sp(2m,C).

These algebras may be defined as follows. Let G be a classical group. Let V be its defining representation, and let T^fV be the f th tensor power of V. (i.e.,) $T^fV = V_1 \otimes V_2 \otimes \ldots \otimes V_f$. The group action of G on V lifts to the diagonal action of G on T^fV defined by $g.(V_1 \otimes V_2 \otimes \ldots \otimes V_f) = (gV_1) \otimes (gV_2) \otimes \ldots \otimes (gV_f)$. Define the commuting algebra $End_G(T^fV)$ of this action to be the algebra of all linear transformations of T^fV which commute with this action of G. In the case of G = Gl (n, C) Schur showed that there is a surjective algebra homomorphism from CS_f onto $End_{Gl(n,C)}(T^f C^n)$, which is an isomorphism for $f \leq n$. The kernel of this homomorphism, gives a complete explanation of the centralizer algebra $End_{Gl(n,C)}(T^f C^n)$.

In 1937, when G = O(n,C) and Sp(2m,C) Richard Brauer defined two algebras $A_f^{(x)}$ and $B_f^{(x)}$ where 'f' is a positive integer and 'x' is a real indeterminate. The surjective algebra homomorphism for the algebras $A_f^{(x)}$ and $B_f^{(x)}$ are constructed as follows:-

$$\phi_f^{(n)}: A_f^{(n)} \longrightarrow End_{o(n,R)}(T^fR^n)$$

$$\chi_f^{(2m)}: B_f^{(2m)} \longrightarrow End\ sp_{(2m,R)}(T^f R^{2m})$$

If n and m are large enough then these homomorphisms are isomorphisms. When these homomorphisms are not an isomorphism then Richard Brauer failed to give the explanation of the kernel of the maps. In order to give a clear explanation of these kernels, Phil Hanlon and David Wales began to study the structure of the algebras $A_f^{(x)}$ and $B_f^{(x)}$ where 'x' is an arbitrary real. The algebras $A_f^{(x)}$ and $B_f^{(-x)}$ are isomorphic to each other. So it was only necessary to study the algebra $A_f^{(x)}$.

The authors were able to describe the radicals of $A_f^{(x)}$ and the matrix ring decomposition of $A_f^{(x)}$ /Rad $(A_f^{(x)})$. Later this problem was reduced to the problem of computing the ranks of certain combinatorially defined matrices $Z_{m,k}(x)$.

1. Defining the matrix $Z_{m,k}(x)$:

The computational problem will be to compute the rank of certain combinatorially defined matrices $Z_{m,k}(x)$ for every complex number x. The determinant of $Z_{m,k}(x)$ is known to be nonzero as a polynomial in x. So the rank of $Z_{m,k}(x)$ is completely determined except at a finite number of values of x. The finite values of x are those x that are the roots of det $(Z_{m,k}(x))$. So the computational problem breaks into two parts:-

1. Compute the roots of det $(Z_{m,k}(x))$.
2. For each root r, compute the rank of $Z_{m,k}(r)$

1.1. Definition:

Let m and k be nonnegative integers. An (m, k) partial 1-factor is a graph with m+2k points and k lines which satisfies:-

1. Every point has degree 0 or 1.
2. The m points of degree 0 are labeled with the numbers 1, 2... m.

For example,

is a (6, 2) factor.

1.2. Notation:

Here we use 'f' to denote m+2k, and lower case Greek letters $\delta, \delta_1, \delta_2, \ldots$ to denote partial 1-factors. $B_{m,k}$ denote the set of all (m, k) partial 1–factors. Let $V_{m,k}$ be the complex vector space with basis $B_{m,k}$. The points of degree '0' in a partial 1-factor δ are called the free points of δ.

1.3. Definition:

Let δ_1 and δ_2 be elements of $B_{m,k}$. The union of δ_1 and δ_2 is a graph consisting of some number $\gamma(\delta_1,\delta_2)$ of cycles together with m paths $P_1 \ldots P_m$. If u is an endpoint of P_i, then u is a free point of either δ_1 or δ_2. Hence, the end points of each path are labeled. We say δ_1 and δ_2 are consistent if each path of $\delta_1 \cup \delta_2$ has the property that its endpoints have the same label. Otherwise, δ_1 and δ_2 are inconsistent.

1.4. Definition:

Let m and k be nonnegative integers. Define a matrix $Z_{m,k}(x)$ with rows and columns indexed by $B_{m,k}$. For $\delta_1, \delta_2 \in B_{m,k}$. let the $(\delta_1,\delta_2)^{th}$ entry of $Z_{m,k}(x)$ be defined by

$$Z_{m,k}(x)_{\delta_1,\delta_2} = \begin{cases} x^{\gamma(\delta_1,\delta_2)} & \text{if } \delta_1 \text{ and } \delta_2 \text{ are,} \\ 0 & \text{if } \delta_1 \text{ and } \delta_2 \text{ are inconsistent.} \end{cases}$$

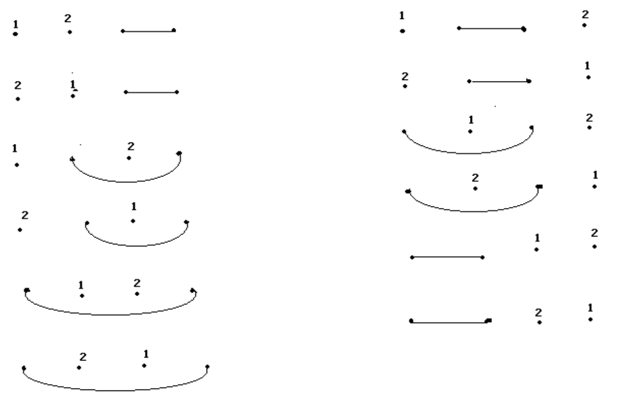
1.5. Note:

Each diagonal entry of $Z_{m,k}(x)$ is x^k and that every off-diagonal entry is either 0 or x^e with e<k. So the determinant of $Z_{m,k}(x)$ is a nonzero polynomial in x of degree $|B_{m,k}|$.

1.6. Example:

Let f=4 and m=2. In this case, the matrix $Z_{m,k}(x)$ is 12 x 12.

An ordered basis for $B_{m,k}$ is given below:



Thus, the matrix $Z_{m,k}(x)$ with respect to this is given by

$$Z_{2,1}(x) = \begin{pmatrix} x & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & x & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & x & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & x & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & x & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & x & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & x & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & x & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & x & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & x & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & x & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & x \end{pmatrix}$$

The size of $B_{m,k}$ is

$$|B_{m,k}| = (m+2k)! / 2^k k!$$

Here m=2, k=1.
for the above the size of $B_{m,k}$ is
$(2+2*1)! / 2^1\, 1! = 4! /2 = 4*3/2 = 6$

1.7. Theorem:

Let G be a finite group with irreducible representation $\Phi_{1,\dots,}\Phi_c$. Let Φ be a representation of G on a complex vector space V which decomposes into irreducible as $\Phi = \Sigma m_i \Phi_i$; i: 1 to c

Let Z be a linear transformation of V which commutes with the action of G. Then Z is similar to a matrix which is a direct sum over i of matrices Z_i, where Z_i is a m_i x m_i matrix repeated in the direct sum deg (Φ_i) times. Moreover, Z_i can be computed as follows:

Step 1: Choose a complete set of Primitive orthogonal idempotent

$e_v^{(u)} : 1 \le u \le c,\ 1 \le V \le \deg(\Phi_u)$ in the group algebra CG.

Step 2: Find m_i vectors $v_1 \dots v_{mi} \in V$ such that $\Phi(e_1^{(i)})\, v_{mi}$ are linearly independent.

Step 3:-

Let $v_1^{(i)}$ be the subspace of V spanned by $\Phi(e_1^{(i)})\, v_1 \dots \Phi(e_1^{(i)})\, V_{mi}$. The space $v_1^{(i)}$ is Z- invariant and Z_i is the restriction of Z to $v_1^{(i)}$.

2. COMPUTING THE BRAUER ALGEBRA DISCRIMINANTS

2.1. Lemma:

Let μ and λ be partitions of m and f respectively, and let m (μ, λ) denote the multiplicity of $\Phi_\lambda \otimes \Phi_\mu$ in $V_{m,K}$ .Then

$$m(\mu, \lambda) = \sum_{\substack{\eta \vdash 2k \\ \eta \text{ even}}} g_{\lambda\mu\eta}$$

Proof:

Let G be S_f x S_m

Let H be the subgroup $(S_{2k}$ x $S_m)$ x S_m , and let S be the subgroup of H given by

$S = (\pi, \sigma, \sigma)$: , $\pi \in B_{2K}, \sigma \in$ Sym(m)

Here, B_{2K} denotes the hyperoctahedral group of k x k signed permutation matrices, which is considered to be a subgroup of S_{2k}.

G acts as a transitive permutation group on the set $B_{m,k}$. So the action of G on $V_{m,k}$ is the induction of the trivial character ε from the stabilizer of any $\Delta_\circ \in B_{m,k}$ to G
Choose
We have the stabilizer of Δ_0 is S.
Using a theorem of Littlewood and some well- known facts

$\Delta_0 =$ 

about the structure of group algebras, we have,

$$\mathrm{ind}_s^H(\varepsilon) = \bigoplus_{\substack{\eta \vdash 2k \\ \eta \text{ even}}} \bigoplus_{\mu \vdash m} \varphi_\eta \otimes \varphi_\mu \otimes \varphi_\mu$$

By the Littlewood – Richardson rule we have for each η,μ.

$$\mathrm{ind}_H^G(\varphi_\eta \otimes \varphi_\mu \otimes \varphi_\mu) = \sum_{\lambda \vdash f} g_{\lambda\mu\eta}\ \varphi_\lambda \otimes \varphi_\mu$$

Hence the theorem is proved.

Now, fix partitions $\mu \vdash m$ and $\lambda \vdash f$.If μ is not cntained in λ, then $g_{\lambda\mu\eta} = 0$
for all η,

So $m(\mu, \lambda) = 0$.

Therefore, we may assume that $\mu \subseteq \lambda$.

For that, we will have to identify a particular idempotent 'e' in the group algebra of G corresponding to the irreducible representation $\varphi_\lambda \otimes \varphi\mu$.

To obtain the idempotent 'e', first let S_0 be the minimal standard young tableau of shape μ.

Therefore,

$$S_0 = \begin{array}{cccc} 1 & 2 & \dots & \mu_1 \\ (\mu_1+1) & (\mu_1+2) & \dots & (\mu_1+\mu_2) \\ (\mu_1+\dots+\mu_{l-1}+1) & & \dots & m \end{array}$$

Next, let t_0 be the standard young tableau of shape λ which agrees with S_0 on the intersection of λ and μ and which has the minimal filling of $[\lambda/\mu]$ with m+1,...,f.

Therefore,

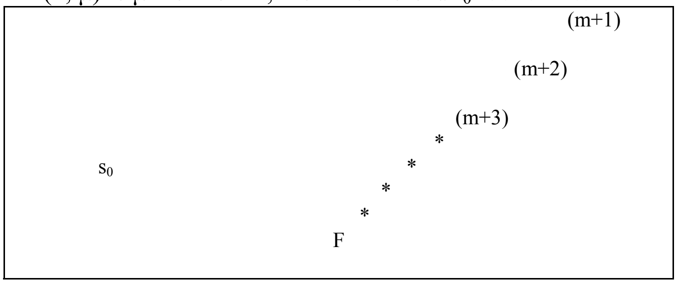
1	2	...	μ_1
$(m+1) \ldots (m+\lambda_1-\mu_1)$			
$t_0 = (\mu_1+1)$	$(\mu+2)$	$\ldots (\mu_1+\mu_2)$	$(m+\lambda_1-\mu_1+1)$
$(\mu_1+\ldots \mu_{l-1}+1)$	...	m	
$(f-\lambda_d+1)$	...	f	

Then 'e' defined by $e = e_{to} x\ e_{so}$ is 'e' is an idempotent in the group algebra of G corresponding to the irreducible $\varphi_\lambda \otimes \varphi\ \mu$.

2.2. Definition:-

The pair (λ, μ) is μ - external if $[\lambda/ \mu]$ has no pair of squares in the same row or the same column.

If (λ, μ) is μ – extremal, then the tableau t_0 looks like.



Any lattice permutation of length 2k and shape η constitutes a littlewood – Richardson filling of $[\lambda/ \mu]$. So for all η we have,

$$g_{\lambda\mu\eta} = f_\eta$$

$$\text{Thus, } m(\mu,\lambda) = \sum_{\eta\text{even}} f_\eta$$

$$= 1.3....(2k-1).$$

Hence the multiplicity $m(\mu,\lambda)$ equals the number of 1-factors on 2k points.

2.3. Note:-

For any pair (λ,μ) we have $g_{\lambda\mu\eta} \le f_\eta$

In general $m(\mu,\lambda) \le 1.3..........(2k-1)$.

Equality is achieved if and only if (λ,μ) is μ -extremal.

2.4. Definition:

Let Δ be a 1- factor with 2k points. Define the (m, k) partial 1- factor V_Δ as follows:

1. V_Δ has free points 1,2............m. The free point j has label j.
2. For every edge (u,v) of Δ we have the edge (m+u, m+v) of V_Δ

The following lemma will not only show that the ev_Δ linearly independent, it will also greatly streamline our computation. This result is difficult to prove in the case of general pairs(λ,μ).

2.5. Lemma:

Let (λ,μ) be μ-extremal and define t_0, s_0 as above. Let γ, σ, γ' and σ' be in C_{to} R_{to} C_{so} and Rs_o, respectively (so, sgn (γ) sgn (γ') $(\gamma\sigma, \gamma'\sigma')$ is one of the terms occurring in the idempotent $e = e_{to}$ x e_{so}). Suppose that.

$(\gamma\sigma, \gamma'\sigma')\ V\Delta = V\Delta_1$, where Δ and Δ_1 are 1-factors. Then

1. $\Delta = \overline{\Delta_1}$,
2. γ and σ both fix t_0 / s_0 point wise.
3. γ restricted to s_0 equals γ' and σ restricted to s_0 equals σ' .

In particular, $\{eV_\Delta$: Δ is a 1- factor with 2k points$\}$ is a basis for $eV_{m,k}$.

Proof:

Let γ' σ' acts on an m,k Partial 1-factor by changing the labels on the free points by (γ', σ').

Since the free points of both Δ and Δ, are 1,2 ,...,m, it follow that $\gamma\sigma$ Preserves the sets { 1,2,...,m} and {m+1,...,f}.

In t_0, each square m+u(u=1,2,...,2k) is at the right-hand end of the row containing it and at the bottom of the column containing it.

So σ moves the point m+u weakly to the left. Since the image of m+u under $\gamma\sigma$ is in the set {m+1,...,f}, the permutation γ must then move σ(m+u) down to the bottom of the column it occupies.

Thus, $(\gamma\sigma)$ (m+u) = m+v, where $v \ge u$.

It follows that $\gamma\sigma$ fixes the set {m+1,..,f} pointwise.

Now, consider $\gamma\sigma$ x γ' σ' on s_0. The point j is moved by $(\gamma\sigma, \gamma'\sigma')$ to $\gamma\sigma$(j) and its label is changed to $(\gamma'\sigma')$j.

Since $(\gamma\sigma, \gamma'\sigma')\ V_\Delta = V_\Delta$, we have $(\gamma\sigma)j = (\gamma'\sigma')\ j$ for all j.

Thus, $\gamma\sigma = \gamma'\sigma^{1}$', so $\gamma = \gamma'$ and $\sigma = \sigma^1$ where these last three equalities refer to $\gamma\sigma$, γ and σ restricted to the points of s_0.

Suppose $\Sigma_\Delta a_\Delta eV_\Delta=0$.Then $\Sigma_\Delta a_\Delta V_\Delta=0$,and so all $a_\Delta=0$,as the V_Δ are linearly independent.

Thus the set $\{ eV_\Delta\}$ is a basis.

2.6. Definition:

Let $(a_i,b_i)(i=1,2,...,2k)$ be the co-ordinates of the squares of $[\lambda\backslash\mu]$.For each i define sets $C^{(i)} \subseteq C_{t0}$ and $R^{(i)} \subseteq R_{t0}$ as follows:

1. $C^{(i)}$ contains the identity permutation as well as the (a_{i-1}) involutions $\gamma_{i,j}$ which exchange the elements of t_0 in squares (a_i,b_i) and (j,b_i)
2. $R^{(i)}$ contains the identity permutation as well as the (b_{i-1}) involutions $\gamma_{i,j}$ which exchange the elements of t_0 in squares (a_i,b_i) and (a_i,j)

Let C be the set of all products $\gamma^{(1)}...\gamma^{(2k)}$,where $\gamma^{(i)} \in C^{(i)}$,and let R be the set of all products $\sigma^{(1)}... \sigma^{(2k)}$ where $\sigma^{(i)} \in R^{(i)}$

C and R are subsets of C_{t0} and R_{t0} of sizes

$|C| = a_1a_2...a_{2k}$, $|R| = b_1b_2...b_{2k}$,

Hanlon and Wales algorithm:

Let G be a finite group with irreducible representation $\Phi_{1,...,}\Phi_c$.Let Φ be a representation of G on a complex vector space V which decomposes into irreducible as $\Phi=\Sigma m_i\Phi_i$;i:1 to c

Let Z be a linear transformation of V which commutes with the action of G. Then Z is similar to a matrix which is a direct sum over i of matrices Z_i, where Z_i is an m_i x m_i matrix repeated in the direct sum $\deg(\Phi_i)$times. Moreover, Z_i can be computed as follows:

Step 1 : Choose a complete set of Primitive orthogonal idempotents

$e_v^{(u)}$: $1 \le u \le c$, $1 \le V \le \deg(\Phi_u)$ in the group algebra CG.

Step 2: Find m_i vectors v_1....... $v_{mi} \in V$ such that $\Phi(e_1^{(i)})\ v_{mi}$ are linearly independent.

Step 3:-

Let $v_1^{(i)}$ be the subspace of V spanned by $\Phi(e_1^{(i)})$ v_1 ... $\Phi(e_1^{(i)})$ V_{mi}. The space $v_1^{(i)}$ is Z- invariant and Z_i is the restriction of Z to $v_1^{(i)}$.

The above algorithm will compute the $V_{\Delta 2}$, $V_{\Delta 1}$ entry in $Z_{\lambda,\mu}(x)$ as a sum of terms of the form $\tau=\{(\gamma\sigma,\gamma'\sigma')\ V_{\Delta 2}, V_{\Delta 1}\}$ where $\gamma\in C,\sigma\in R$, $\gamma'\in C_{s0},\sigma'\in R_{s0}$. For a fixed pair (r, σ) Є C X R there is atmost one pair (r' , σ') Є C_{so} x R_{so} for which τ is nonzero.

We next write down a method for computing π= Y' σ' given (Y,σ) $V_{\Delta 2}$, and $V_{\Delta 1}$, In the description below we will assume that the in put is δ1= Y σ $V_{\Delta 2}$ and $\delta_2 = V_{\Delta 1}$.

2.7. Definition:

Let δ1 and δ_2 be (m, k) partial 1- factors. Define an element π (δ1,δ_2) in the group algebra CS_m according to the following algorithm.

For each i in the set { 1,2,...,m} find the unique path in δ1 υδ_2 which begins at the free point of δ1 labelled i and ends at some other free point y. If y is a free point of δ1, then π(δ1,δ_2)=0 and algorithm stops. Otherwise, y is a free point of δ_2.Let π (δ1,δ_2)(i) be the label on y.

When this algorithm finishes, we will have either, π(δ1,δ_2) = 0 or else π(δ1,δ_2) Є S_m. For δ1,δ_2 both (m, k) partial 1- factors and r a standard young tableau of size m, define Γ_Y(δ1,δ_2) by

$$\Gamma_Y(\delta 1,\delta_2) = \begin{cases} 0 & \text{if } \pi(\delta 1,\delta_2) = 0 \text{ the coefficient of } \pi(\delta 1,\delta_2) \text{ in the young symmetrizer} \\ e_r & \text{if } \pi(\delta 1,\delta_2) \in S_m \end{cases}$$

2.8. Lemma:

Let Y = $Y^{(1)}$ ... $Y^{(2k)}$ be in C and σ = $\sigma^{(1)}$ .... $\sigma^{(2k)}$ are not identity. Then Γs_o(Y σ $V_{\Delta 2}$, $V_{\Delta 1}$) = 0

Proof:-

Fix i such that $Y^{(i)}$ =(u, b_i) with u<a_i and $\sigma^{(i)}$ = (a_i,v)with V< b_i.

Let α and β be the labels in the squares (u, b_i) amd (a_i,V) of t_o.

The row permutation σ moves the label β to the corner square (a_i, b_i).

Then the column permutation Y moves the label to the square (u, b_i).

So in Y σ $V_{\Delta 2}$ υ $V_{\Delta 1}$, the path beginning at the free point labeled β in Y σ $V_{\Delta 2}$ has length 0 and ends at the same point of $V_{\Delta 1}$ which is a free point labeled α.

So, π (Y σ $V_{\Delta 2}$, $V_{\Delta 1}$) (β) = α.

But the corner Square (a_i, b_i) does not exist in s_o, so π(Y σ $V_{\Delta 2}$, $V_{\Delta 2}$) moves β from position (a_i,V) to (U, b_i) where b_i > μa_i.

It is easy to see that such a permutation cannot be written in the form Y σ where

Y Є ~~C_{so} and~~ σ Є R_{so}.

So, (ΓS_o (Y σ $V_{\Delta 2}$, $V_{\Delta 1}$,)=0.

This couplets the proof.

2.9. Note:

Let S denote the set of Pairs (Y σ) with Y=$Y^{(1)}$... $Y^{(2k)}$ Є C and σ = $\sigma^{(1)}$ .... $\sigma^{(2K)}$ Є R Such that $\sigma^{(1)}$ is the identity wherever $Y^{(i)}$ is not the identity.

The size of S is ∣S∣= π(ai+bi-1).

2.10.Theorem:

The following algorithm compute the Δ_i,Δ_j entry in $Z_{\lambda,\mu}(x)$.

Algorithm for each pair (γ,σ)∈δ.

1.Compute Γ_{S0} (γσ$V_{\Delta j}$,$V_{\Delta i}$).

2.Compute the number of cycles N in γσ$V_{\Delta j}$U$V_{\Delta i}$.

3.Add Sgn(γ)Γ_{S0} (γσ$V_{\Delta j}$,$V_{\Delta i}$)x^N to the current value of $Z_{\lambda,\mu}(x)$.

Before proving this algorithm consider the case λ=(6,5,4,3,2,1) and μ=(5,4,3,2,1).

The size of original matrix $Z_{m,\mu}(x)$ is a whopping (21)!/48. The submatrix $Z_{\lambda,\mu}(x)$ is 15x15. The six squares of [γ/μ] have

co-ordinates (1,6),(2,5),(3,4),(4,3),(5,2) and (6,1), so the size of S is 6^6.

Thus each entry of $Z_{\lambda,\mu}(x)$ is computed with 6^6 passes through the main loop of the algorithm in above theorem.

In practice,this matrix $Z_{\lambda,\mu}(x)$ was computed in about one hour of CPU time on a CRAY-2.

In general,we must perform the main loop in the above algorithm π(a_i+b_i-1) times.This main loop is carried out in O(f+∑a_j(a_j+1))steps.

So, the above Theorem gives a method to compute each entry of $Z_{\lambda,\mu}(x)$ in 0{ π(a_i+b_i-1) (f+∑a_j(a_j+1)}steps.

Proof:

Let e= $c_{s0}r_{s0}$ be the young symmetrizer indexed by s_0.

According to Theorem, Let G be a finite group with irreducible representation $\Phi_{1,...,}\Phi_c$.Let Φ be a representation of G on a complex vector space V which decomposes into irreducible as Φ=Σm¡Φ_i;i:1to c

Let Z be a linear transformation of V which commutes with the action of G. Then Z is similar to a matrix which is a direct sum over i of matrices Z_i, where Z_i is an m_i x m_i matrix repeated in the direct sum deg(Φ_i)times. Moreover, Z_i can be computed as follows:

Step 1 : Choose a complete set of Primitive orthogonal idempotents

$e_v^{(u)}$: 1≤ u ≤c, 1≤ V ≤deg(Φ_u) in the group algebra CG.

Step 2: Find m_i vectors v_1....... v_{mi} εV such that $\Phi(e_1^{(i)})$ v_{mi} are linearly independent.

Step 3:-

Let $v_1^{(i)}$ be the subspace of V spanned by $\Phi(e_1^{(i)})$ v_1 ... $\Phi(e_1^{(i)})$ V_{mi}. The space $v_1^{(i)}$ is Z- invariant and Z_i is the restriction of Z to $v_1^{(i)}$.

the matrix $Z_{m,k}(x)$ preserves the subspace {ev_Δ:Δ is a 1-factor of size 2k}.By Lemma, Let (λ,μ) be μ-extremal and define t_0, s_0 as above. Let γ, σ, γ' and σ' be in C_{to} R_{to} C_{so} and Rs_o, respectively (so, sgn (γ) sgn (γ') (γσ, γ'σ') is one of the terms occurring in the idempotent e = e_{to} x e_{so}). Suppose that.

(γσ, γ'σ') V$\underline{\Delta}$ = VΔ_1, where Δ and Δ_1 are 1-factors. Then

4. Δ = Δ_1,

5. γ and σ both fix t_0 / s_0 point wise.

6. γ restricted to s_0 equals γ' and σ restricted to s_0 equals σ' .

In particular, {eV_Δ : Δ is a 1- factor with 2k points} is a basis for $eV_{m,k}$.

the co-oeficient of $v_{\Delta i}$ in $ev_{\Delta j}$ is 0 for i≠j and is∣R_{s0}∣∣C_{s0}∣for i=j.

so the i,j entry of $Z_{\lambda,\mu}(x)$ is $(1/|R_{s0}||C_{s0}|)$ times the co-efficient of $v_{\Delta i}$ in $Z_{\lambda,\mu}(x)(ev_{\Delta j})$.

Thus

$$(Z_{\lambda,\mu}(x))_{\Delta i,\Delta j} = \frac{1}{|R_{s0}||C_{s0}|}(ev_{\Delta j}, v_{\Delta i})$$

$$= \frac{1}{|R_{s0}||C_{s0}|} \sum_{\substack{\gamma\in C_{t0} \\ \sigma\in R_{t0}}} Sgn(\gamma)$$

$$= \sum_{\substack{\gamma'\in C_{s0} \\ \sigma'\in R_{s0}}} Sgn(\gamma')\{(\gamma\sigma,\gamma'\sigma')\, V_{\Delta i,}$$

$$= \sum_{\gamma\in c} \sum_{\sigma\in R} Sgn(\gamma)\lceil_{S0}\ (\gamma\sigma V_{\Delta j},V_{\Delta i})\,(\gamma\sigma V_{\Delta j},V_{\Delta i}).$$

This equality follows from the definition $\lceil_{S0}$. we have

$$(Z_{\lambda,\mu}(x))_{\Delta i,\Delta j} = \sum_{(\gamma,\sigma)\in S} Sgn(\gamma)\lceil_{S0}\ (\gamma\sigma V_{\Delta j},V_{\Delta i})\,(\gamma\sigma V_{\Delta i},V_{\Delta i}).$$

This completes the proof.

CONCLUSION:

I have tried to give a brief sketch of some of the main ideas underlying the dynamically growing field of Brauer Centralizer Algebra. It is the natural Convergence of ideas from many areas of mathematics such as algebra, combinatorics, with those from computers science, such as algorithms, data structures. I feel confident that the current trend of studying Brauer Algebra will continue to suggest new classes of problems which are can continue for further enrichment of his knowledge.

BIBILIOGRAPHY:-

1. R.Brauer, On algebras which are connected with the semisimple continuous groups ann of math.(2) 38(1937), 857-872.
2. P.Hanlon and D.B. Wales, On the decompositron of Brauer's centralizer algebras, J. Algebra 121 (1989), 409-445.
3. P.Hanlon and D.B.Wales, Eigen values connected with Brauer's centralizer algebras, J.Algebra 121 (1989), 446-476.
4. IG.Macdonald, Symmetric functions and Hall polynomials, Oxford univ. press, London, 1979.
5. H.wenzl, On the structure of Brauer's centralizer algebras, Ann of math, 128(1988), 173-193.
6. I.S Luthar, I.B.S. passi, Algebra-Volume 3, Modules, Narosa Publishing house, 2005.
7. P.Hanlon and D.Wales, computing the discriminants of Brauer's centralizer algebras, math comput. 54. No. 19(1990), 77-0796.